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Capacitated Stochastic Fractional Transhipment Promlem

ABSTRACT

The present paper analyses a transshipment type fractional programming under stochastic demand. The objective is to maximize the expected profitability of the total transshipment schedule, the net expected profitability being defined as the total expected revenue minus the loss in transshipment. The stochastic transshipment problem is reduced to an equivalent deterministic transportation problem for which an algorithm is developed and numerically illustrated

1. INTRODUCTION

Orden [7] proposed a generalized transportation model in which transshipment through intermediate points is permitted. In this paper transshipment problem that often occurs in the distribution system of the national department store chain is considered, treating the demands as uncertain. To evaluate the performance of an economic activity, "profitability" (i.e. the ratio of the profit earned to the costs incurred) is sometimes regarded as a better indicator than the net profit. In this paper, we study the stochastic linear fractional programming problem in which the parameters of only the numerator of the fractional objective functional are treated as random variables while the parameters of the denominator are assumed to be fixed. The purpose of this paper is to develop an algorithm for a capacitated transshipment problem in which the demands are random. The objective is to maximize the expected profitability of the transshipment schedule. The algorithm developed for the resulting deterministic problem, itself is the outcome of the basic result that for linear fractional programming the absolute minimum occurs at a basic feasible solution. The technique applied by Ferguson and Dantzig [1] are used for dealing the problems under stochastic environment.

2. NOTATIONS AND THE FORMULATION OF THE PROBLEM

We consider a transshipment problem with m sources and n sinks numbered as 1, 2, … ,m and n sinks numbered as m+1, m+2,…., m+n. Let,

a_i = the quantity available at source i = 1,2,……..m,

d_j = the quantity demanded at sink j = m+1,m+2,………….,m+n,

x_{ij} = the quantity shipped from station i to j (i, j = 1,2,……,m+n),

c_{ij} = the per unit shipment cost from station i to j (i, j = 1,2,……,m+n),

u_i = quantity transshipped at the station i (i = 1,2,……,m+n),

c_i = per unit transshipment cost (including unloading, reloading, and storage etc.) at the station i (i = 1,2,……,m+n), The problem is to determine x_{ij} so as to minimize the total cost of transportation and transshipment. It may be mathematically stated as under:

Problem P_1: Find x_{ij} so as to

$$\text{Minimize } F = \sum_{i=1}^{m+n}{}^{*} \sum_{j=1}^{m+n}{}^{*} c_{ij} x_{ij} + \sum_{i=1}^{m+n} c_i u_i \quad ....(2.1)$$

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subject to =

...(2.2a)
..(2.2)
...(2.2b)

=

...(2.3a)
...(2.3)
...(2.3b)

x_{ij} 0 for all i & j …(2.4)

indicates that the term j = i is

excluded from the sum. The constraints (2.2a) and (2.2b) implies that the total quantity that leaves the source i (= 1, 2, …, m) is equal to the quantity available plus the quantity transshipped and the total quantity that leaves the sink i (= m+1, m+2, …, m+n) is equal to the quantity transshipped. Similarly constraints (2.3a) and (2.3b) implies that the total quantity that arrives at a source i (= 1, 2, …, m) is equal to the quantity that source transships and the total arriving at a sink is equal to the demand at that sink plus the quantity that the sink transships. Constraints (2.4) is the usual nonnegative restrictions. Here u_i are unknown, so we impose an upper bound u_o (say), on the amount that can be transshipped at any point, so that

$u_i = u_o - x_{ii}$, i = 1, 2, …, m …(2.5)

where, x_{ii} is a nonnegative slack. After substituting (2.5) in (2.1) to (2.3) and on simplifying the original transshipment problem P_1 is reduced to the following genuine transportation type linear programming problem.

Problem P_2:

Minimize F =

subject to

...(2.6a)
...(2.6b)
....(2.6)

...(2.7a)
...(2.7b)
…(2.7)

x_{ij} 0 for all i & j …(2.8)

where $c_{ii} = -c_i$, and the asterisk (*) on the summations has now been disappeared. As $u_i \geq 0$, we must have $x_{ii} \leq u_o$ which is guaranteed by equations (2.6) – (2.7), because any x_{ii} will always appear in one equation that has u_o on the right hand side. Here, the upper bound u_o can interpreted as the size of a fictitious stockpile at each source and sink which is large enough to take care of all transshipments. Assume initially a value for u_o which is sufficiently large to ensure that all x_{ii} will be in the optimal basis. Such a value can be easily found as the volume of goods transshipped at any point cannot exceed the total volume of goods produced (or received). Hence, set,

$u_o =$ ….(2.9)

which ensures that u_o is not limiting. The unused stockpile at the station i = 1, 2, ……. m+n, if any, will be absorbed in the slack x_{ii}.

3. PROBLEM REFORMULATION UNDER STOCHASTIC ENVIRONMENT [2, 6]

Till now, we have treated the demand d_j as uncertain as if they are fixed constraints. However, for our study, we assume d_j as an independent discrete random variable with known probability distributions. Here A_{ij} is the additional upper bound restrictions on route capacities (i, j). Let d_{ij} be the per unit procurement cost of the product at origin i plus the per unit loss due to pilferage etc. on the route (i, j); r_j and

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s_j are respectively the revenue received (e.g. sale proceeds) and handling costs (e.g. seller's commission etc.) for each unit of demand satisfied at destination j; and are yet unknown functions representing respectively the total expected revenue and the total expected handling costs at destination j if a total of y_j unit is shipped to this destination. The objective function is to maximize the ratio of the net expected revenue to the total cost incurred. Thus our problem is of the following form:

Problem P_3: Max. Z =

…(3.1)

subject to

$$\sum_{j=1}^{m+n} x_{ij} = \begin{cases} a_i + u_o & i = 1, 2, ..., m \\ u_o & i = m+1, ...m+n \end{cases} \quad …(3.2)$$

$$\sum_{i=1}^{m+n} x_{ij} = \begin{cases} u_o & j = 1, 2, ..., m \\ d_j + u_o & j = m+1, ...m+n \end{cases} \quad …(3.3)$$

$$0 \le x_{ij} \le A_{ij} \qquad (\forall\, i,\, j) \qquad ...(3.4)$$

Here, the third term of the denominator viz. $\sum_{i=1}^{m+n} c_i u_o$ is a constant.

The maximization of the above Problem P_3 is obviously, equivalent to maximizing:

Expected Profitability

$$= \frac{\text{Expected Net Revenue - Total Cost}}{\text{Total Cost}}$$

$$= \frac{\text{Expected Net Revenue}}{\text{Total Cost}} - 1 \qquad ...(3.5)$$

Due to the presence of the capacity restrictions, cases may arise in which the problem has no feasible solution. However, for the present study we assume that

a) the set of feasible solutions is regular, and
b) the denominator of the objective functional Z is always strictly positive.

4. THE EQUIVALENT DETERMINISTIC PROBLEM [2, 5]

Let the demand d_j's at various destinations be independent random variables and the probability distribution of d_j (j = 1, 2,…, n) be in increasing order as follows:

Demand d_j $\quad d_{1j} < d_{2j} < \quad d_{H_jj}$

Prob.($d_j = d_{hj}$) = p_{hj} $\quad p_{1j} \quad p_{2j} \quad p_{H_jj}$

Prob. ($d_j \quad d_{hj}$) = π_{hj} $\quad \pi_{1j} = \qquad \pi_{2j} =$

..... $\quad \pi_{H_jj} = p_{H_jj}$

To determine the functions $\phi_j(s_j, y_j)$ and $_j(r_j, y_j)$ note that y_j, the net quantity shipped to sink j, can be any amount between the lowest value d_{1j} and the highest value in the probability distribution of the demand d_j (j = 1, 2, …n).

If then each of the y_j units shall be absorbed with probability π_{1j} (= 1).

Hence, the expected revenue is .

If $d_{1j} \le y_j \quad d_{2j}$, then each unit upto d_{1j} shall be absorbed with probability and each of the additional units (y_j - d_{1j}) shall be absorbed with probability

Hence, the expected revenue is =

So, in general, if $d_{hj} \le y_j \quad d_{h+1,j}$, then the expected revenue is

Let us now break y_j into incremental units y_{hj} (h = 1, 2,…H_j) as:

$$y_j = y_{1j} + y_{2j} + y_{3j} + \ldots\ldots + y_{hj} + \ldots \quad \ldots(4.1)$$

where,

..(4.2)

Relation (4.1) makes physical sense only if there exists some $h = h_j$ (say) such that all intervals below the h_j^{th} interval are filled to capacity and all intervals above it are empty i.e.

...(4.3)

Assuming for the time being that the conditions (4.3) holds, then the total expected revenue and the total expected handling cost at destination j are:

$$\left.\begin{array}{ll} 0 \le y_{1j} \le d_{1j} = F_{1j}\,(say) & y_{hj} = F_{hj} \quad (h = 1, 2, \ldots, h_j - 1) \\ \phi_j(r_j, y_j) = \sum_{h=1}^{H_j} r_j \pi_{hj} y_{hj} & \\ 0 \le y_{2j} \le d_{2j} - d_{1j} = F_{2j}\,(say) & 0 \le y_{hj} \le F_{hj} \quad (h = h_j) \\ \vdots & y_{hj} = 0 \quad (h = h_j + 1, \ldots, H_j) \\ 0 \le y_{H_j j} \le d_{H_j j} - d_{H_j - 1, j} = F_{H_j j}\,(say) & \end{array}\right\} \text{and} \quad \ldots(4.4)$$

$$\varphi_j(s_j, y_j) = \sum_{h=1}^{H_j} s_j \pi_{hj} y_{hj} \quad \ldots(4.5)$$

After putting the value of $\phi_j(r_j, y_j)$ and $\varphi_j(s_j, y_j)$ from (4.4) and (4.5), the deterministic equivalent of Problem P_3 is of the following form Problem P_4:

Problem P_4: $$\underset{x_{ij}}{Max}\ Z = \frac{\sum_{j=1}^{n}\sum_{h=1}^{H_j} \tau_{hj} y_{hj}}{\sum_{i=1}^{m}\sum_{j=1}^{n} \sigma_{ij} x_{ij}} \quad \ldots(4.6)$$

(where, $\tau_{hj} = (r_j - s_j)\pi_{hj}$ and

$\sigma_{ij} = (c_{ij} + d_{ij})$)

subject to (2.6) and (2.7a)

$$\sum_{i=1}^{m+n} x_{ij} - \sum_{h=1}^{H_j} y_{hj} = 0 \quad j = 1, 2, \ldots, n \quad \ldots(4.8)$$

$$x_{ij},\ y_{hj} \ge 0 \quad \ldots(4.9)$$

...(4.10)

$(\forall\ h,\ j)$ ...(4.11)

and subject to the additional stipulation that the constraints (4.3) are also satisfied.

Fortunately, it turns out that (4.3) do not restrict our choice of optimum solution in any way. This we have proved in the theorem, Javaid et. al. [2].

5. PRELIMINARIES TO THE SOLUTION OF PROBLEM P_4

1. It is assumed that the set of all feasible solutions of Problem P_4 is regular (i.e. non-empty and bounded) and that the denominator of the objective functional is positive for all feasible solutions.[4]

2. The special structure of Problem P_4, permits us to arrange it into an array in Table (5.1).

3. Problem P_4 is a transportation type linear fractional programming problem with upper bound restrictions on some variables; therefore its global maximum exists at a basic feasible solution of its constraints, [5].

4. We shall, hereinafter, call the constraints (4.7) through (4.10) as the original system and the constraints (4.7) through (4.11) as the capacitated system. As none of the constraints in the original system is redundant, a basic feasible solution to the original system shall contain (m+n) basic variables. For the capacitated system also, a basic feasible solution shall contain (m+n) basic variables and the same may be found by working on the original system provided that some of the nonbasic variables are allowed to take their upper bound values [2].

TABLE (5.1)

SPECIAL STRUCTURE OF PROBLEM

The number of rows in the above table is (m+H) where $H = \max H_j$. Obviously, there shall be some empty boxes near the bottom of the table, which may be crossed out.

Absence of the total column below the double line in the above table, indicates that there are no row equations for y_{hj} variables. To obtain the column equations (4.8), each y_{hj} has to be multiplied by (-1). We have omitted (-1) from y_{hj} boxes for convenience.

6. INITIAL BASIC FEASIBLE SOLUTION AND OPTIMALITY CRITERIA

To start with, we fix the demands d_j's approximately equal to their expected values such that

$$\sum_{j=m+1}^{m+n} d_j = \sum_{i=1}^{m} a_i$$

and also such that for all j except $j = j^*$, each d_j falls at the upper end of one of the intervals y_{hj} into which d_j has been divided i.e.

for some and for all j except $j = j^*$ (the d_j can always be so chosen that it is done).

With these fixed demands the upper portion of the Table (5.1) resembles a (m+n) $\times$ (m+n) standard transportation problem for which an initial basic feasible solution with {2(m+n)-1} basic variables may be obtained as follows:

Ignore the upper bounds on x_{ij}'s and write down the basic feasible solution by the North-West Corner Rule or any other method for standard transportation. If this solution satisfies the upper bound constraints, we hit the target. If it violates these constraints, however, then we divide the basic variables into two groups -

(a) the infeasible variables which violate their upper bounds; and

(b) the feasible variables which do not violate them.

Now, we discard temporarily the upper bounds on the infeasible variables and replace the original objective function by one that minimizes the sum of the infeasible variables. The existing solution now acts as the initial basic feasible solution for the artificial problem we have just created, and we begin the iterations, keeping in mind the upper bounds on the feasible variables.

As we proceed, some infeasible variables will increase while others will decrease, but their general level decreases because we decrease their sum. At certain iteration, as soon as some of the originally infeasible variables dip below or become equal to their upper bounds, these variables join the group of feasible variables, become upper bounded and are removed from the objective function. We continue this till

(a) all the infeasible variables disappear or

(b) the objective function cannot be further improved while some infeasible variables still remain.

The later indicates that no feasible solution of the capacitated system exists while the former indicates that a basic feasible solution has been found.

After a basic feasible solution with {2(m+n)-1} variables has been found for the transportation problem (represented in the upper portion of Table (5.1)), we enter in each column of the lower portion of the Table (5.1), non basic y_{hj}'s at their upper bounds in turn h = 1, 2,…. until we have entered enough non basic y_{hj} so that their sum over h is equal to d_j (fixed earlier).

Obviously, we shall never have to enter y_{hj} below its upper bound except in column $j = j^*$, where the last nonzero entry will be This last entry and the {2(m+n)-1} basic x_{ij} found earlier, constitute the required initial basic feasible solution with 2(m+n) basic variables. In case, the last non zero entry in column j^* is also at its upper bound, then we take the last y_{hj} entry of any column as or $2(m+n)^{th}$ basic variable.

7. OPTIMALITY CRITERIA

Let the simplex multipliers corresponding to the objective function

$$Z_1 = \sum_{j=1}^{n} \sum_{h=1}^{H_j} \tau_{hj} y_{hj} \text{ be } \mu_i \ (\forall\, i = 1, 2, ..., m)$$

and v_i $(\forall\ j = 1, 2, ..., n)$ and corresponding to the objective function

$$Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_{ij} x_{ij} \text{ be } \bar{\mu}_i \ (\forall\, i = 1, 2, ..., m)$$

and $\bar{v}_j$ $(\forall\ j = 1, 2, ..., n)$.

These are determined by solving the following equations.

$$\left.\begin{aligned} \mu_i + v_j &= 0 \quad \text{for basic } x_{ij} \\ \tau_{hj} - v_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \qquad ...(7.1)$$

$$\left.\begin{aligned} \bar{\mu}_i + \bar{v}_j &= 0 \quad \text{for basic } x_{ij} \\ -\bar{v}_j &= 0 \quad \text{for basic } y_{hj} \end{aligned}\right\} \qquad ...(7.2)$$

Each of the systems (7.1) and (7.2) involves (m+n) equations in as many as unknowns, and and may be shown to be triangular or atleast semi-triangular, so that both of these systems are easily solvable.

Let the relative cost coefficients corresponding to the variables x_{ij} and y_{hj} be and for the objective function Z_1 and

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and for the objective function Z_2.

These are determined by solving the following equations.

...(7.3)

...(7.4)

For a given basic feasible solution (x_{ij}, y_{hj}), the value of the objective functional Z is:

(say) 7.5

But, the relative cost coefficients corresponding to the for basic variables and also the values of the non basic x_{ij} are zero. As regards the values of non basic y_{hj}'s, some of them are zero and the others at their upper bounds, Fs_{hj}'s. Hence,

$$\frac{\sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*}\lambda_{ij}A_{ij}+\sum_{j=1}^{m+n}{}^{*}\sum_{h=1}^{H_j}{}^{*}\eta_{hj}F_{hj}-\left\{\sum_{i=1}^{m+n}\mu_i(a_i+t_o)+\sum_{i=1}^{m+n}\nu_j t_o\right\}}{\sum_{i=1}^{m+n}{}^{*}\sum_{j=1}^{m+n}{}^{*}\bar{\lambda}_{ij}A_{ij}+\sum_{j=1}^{m+n}{}^{*}\sum_{h=1}^{H_j}{}^{*}\bar{\eta}_{hj}F_{hj}-\left\{\sum_{i=1}^{m+n}\mu_i(a_i+t_o)+\sum_{i=1}^{m+n}\nu_j t_o\right\}}=\frac{Z_1}{Z_2}$$

...(7.6)

where $\sum *$ indicates sum over those non basic y_{hj}'s which are at their upper bounds. Differentiating (7.6) partially w.r.t. the nonbasic variables (x_{ij}, y_{hj}), we get,

and

$$\frac{\partial Z}{\partial y_{hj}}=\frac{(\eta_{hj}Z_2-\bar{\eta}_{hj}Z_1)}{(Z_2)^2}$$

Defining
$$\left.\begin{aligned}\Delta_{ij}&=\lambda_{ij}Z_2-\bar{\lambda}_{ij}Z_1\\ \bar{\Delta}_{hj}&=\eta_{hj}Z_2-\bar{\eta}_{hj}Z_1\end{aligned}\right\}\quad ...(7.7)$$

We observe that the value of Z can be improved in two possible ways by:

* increasing the nonbasic x_{ij} (or y_{hj}) whose (or) are positive

* decreasing those nonbasic x_{ij} (or y_{hj}) whose (or) are negative.

Thus a basic feasible solution is optimum iff

...(7.8)

If any of the optimality criteria (7.8) is violated, the current solution can be improved. The nonbasic variable which violates (7.8) most severely is selected to enter the basis. The values of the new basic variables are found in the usual manner by applying -adjustments. It should, however, be kept in mind that the coefficient of each y_{hj} in the column equations (4.8) is (-1).

The variable to leave the basis is the one that becomes either zero or equal to its upper bound. If two or more basic variables reach zero or their upper bounds simultaneously then only one of them becomes nonbasic. Should it happen that the entering variable itself attains upper or lower bound (zero) without simultaneously making any of the basic variables zero or equal to its upper bounds, the set of basic variables remains unaltered; only their values are changed to allow the so-called entering variable to be fixed at its upper or lower bound.

8. ALGORITHM OF THE DETERMINISTIC PROBLEM

The step-by-step computational algorithm for determining the optimum solution is given as follows:

Step 1. First of all calculate initial/improved basic feasible solution and record them in a working table.

Step 2. Then obtain the values of simplex multipliers and and relative cost coefficients by given equations from equations (7.1) & (7.2) and (7.3) & (7.4) and record them in current working table.

Step 3. Calculate the value of the objective function Z by the equation (4.6).

Step 4. Then for the non-basic variables, calculate and and test whether the solution is optimum or not. If yes, the process terminates and if not, proceed to find the (or) which violates the optimality criteria (7.8), most severely.

Step 5. Find the entering variable as the one whose corresponding (or) violates the optimality criteria most severely.

Step 6. Apply adjustments and determine the outgoing variable (if any) and find the maximum value

Step 7. Go to step 1.

9. NUMERICAL EXAMPLE

Consider the stochastic Transportation problem involving Transshipment with 3 origins and 2 destinations in which shipping charges (costs from origin to origin, destination to destination and origin to destinations) denoted by d_{ij} and c_{ij} are given in the form for each boxes in Table (8.1), Table (8.2) to Table (8.3) respectively. In Table (8.4), the initial basic feasible solution by the North-West Corner is given.

Table (8.1) **Table (8.2)**

d_{ij} / c_{ij}	I (21)	II (4)	III (3)
I	0 0	3 1	1 2
II	3 1	0 0	2 4
III	1 2	2 4	0 0

	A (21)	B (3)
A	0 0	1 3
B	1 3	0

The value of are calculated as follows:

$= d_{ij} + c_{ij}$ and are written in Table (8.4) along with its initial basic solution by North-West Corner Rule of the transportation problem.

Table (8.3)

	I	II	III
I	21 0 0	4 3 1	3 1 2
II	4 3 1	21 0 0	5 2 4
III	3 1 2	5 2 4	21 0 0

Table (8.4)

	A	B
A	21 0 0	3 1 3
B	3 1 3	21 0

In order to determine an optimum solution for stochastic transportation problem, we first calculate and R_{hj} and write down the equivalent deterministic Problem P_4 in the form of Table (8.5).

Table (8.5) **Probability Distribution of demand**

j	b_j	p_{bj}		$\tau_{hj} = (r_j - s_j)\pi_{hj}$	R_{hj}
1	9	0.2	1.0	10.0	9
	12	0.6	0.8	8.0	3
	16	0.2	0.2	2.0	5
2	7	0.2	1.0	5.0	7
	10	0.8	0.8	4.0	3

$\boxed{21}$	0 5	0 3	$\boxed{8}$ - θ	$\boxed{2}$ + θ	31	-4	-1
0 3	$\boxed{21}$	0 5	4 0 –1.5	$\boxed{1}$	26	-4	-2
0 3	0 7	$\boxed{21}$	0 M+3	$\boxed{6}$	27	-4	-1
0 6	0 6.5	0 M+3	$\boxed{21}$	0 6	21	-4	2
0 2	0 4	0 2	0 2	$\boxed{21}$	21	-4	0
Iteration - I			9 $\boxed{-50}$ 6 2	7 1 0			
			3 - θ $\boxed{-138}$ 4 2	$\boxed{2}$ + θ			
			-2 2				
$\nu_1 = 4$	$\nu_2 = 4$	$\nu_3 = 4$	$\nu_4 = 4$	$\nu_5 = 4$			
$\bar{\nu}_1 = 1$	$\bar{\nu}_2 = 2$	$\bar{\nu}_3 = 1$	$\bar{\nu}_4 = -2$	$\bar{\nu}_5 = 0$			

Table 8.7

Deterministic Version of Problem P_4 along with the calculated values of and are given in Table(8.6) a_i

Iteration-1

Step-1. In order to obtain initial basic feasible solution we fix the demands at $b_1 = 12$ and $b_2 = 9$, and then determine a starting basic feasible solution to the (3 2) standard transportation problem represented in Table (8.4), by the North-West corner Rule.

Then a standard transshipment problem can be formed (ignoring the upper bounds). We get,

$x_{11} = 21$, $x_{14} = 10$, $x_{15} = 2$,

$x_{22} = 21$, $x_{24} = 4$, $x_{25} = 1$,

$x_{33} = 21$, $x_{35} = 6$, $x_{44} = 21$,

$x_{55} = 21$

This solution violates the upper bound constraints as

To obtain a basic feasible solution to the deterministic capacitated transshipment problem, we temporarily treat all x_{ij}'s, except the infeasible variable x_{24}, as upper bounded and apply the usual transportation routine to minimize the sum of infeasible variable, i.e., to minimize x_{24}, till the infeasibility of x_{24} is removed. The solution so obtained is:

$x_{11} = 21$, $x_{14} = 8$, $x_{15} = 2$,

$x_{22} = 21$, $x_{24} = 4$, $x_{25} = 1$,

$x_{33} = 21$, $x_{35} = 6$, $x_{44} = 21$,

$x_{55} = 21$

For the capacitated transshipment problem $x_{24} = 4$ is non basic variable at its upper bound. Now in each column of the working tables we assign values to y_{hj} variables to their upper bounds (as far as possible).

We get $y_{11} = 9$, $y_{21} = 3$, $y_{12} = 7$ and $y_{22} = 2$ $(< R_{22})$.

Step-2. The simplex multipliers and relative cost coefficients are determined from equations (7.1) and (7.2). These are also recorded in the working Table (8.7).

Step-3. The values of Z are obtained using equations (7.6): $Z_1 = 157$ and $Z_2 = 44$ and finally $Z = \frac{Z_1}{Z_2} = \frac{157}{144} = 3.56.$

Step 4. For the non-basic variables, the values of Δ_{ij} and are calculated and found that and violets the optimality criteria.

= 6(44) – 2(157) = 264 –314 = -50 and

= 4(44) – 2(157) =176 - 314 = -138

But violates most severely, obviously the current solution is not optimum and may further improved.

Step-5. Adding to we are led to the -adjustments as shown in Table (8.7). The maximum possible value of is

After **two iterations** the optimal solution has been attained as:

$Z_{opt} = 3.64$

$x_{11} = 21$, $x_{14} = 7$, $x_{15} = 3$,

$x_{22} = 21$, $x_{24} = 4$, $x_{25} = 1$,

$x_{33} = 21$, $x_{35} = 6$, $x_{44} = 21$,

$x_{55} = 21$

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