

Estimation of Odds Ratio as A Quality Indicator on Investment Recommendations – A Bayesian Approach



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ABSTRACT

A Stock brokerage is a service-oriented agency whose primary objective is to buy or sell shares on behalf of their clients and they also deal with broking services, research, wealth management, retirement planning, depository services, mutual funds, etc., This study is about finding a pattern of profit and loss in two types of call recommendations (Buy/Sell) based on the data collected from a reputed stock brokerage firm. The inherent advantage in handling Bayesian modelling has been attempted with the necessary models using suitable transformation of underlying parameters. Quantifying the measure of associations between the variable of interest is achieved through odds ratio together with the measure of heterogeneity. Various models could be achieved through possible combination of variables and the results are presented both in numerical and graphical mode. This study has made an attempt in building a model based on the recommendations of a stock broker. Based on the data received from the stock broker, the response metric variable is treated as a categorical variable using appropriate rules. Identifying suitable associated variables to understand the variability quantification in a more better way and the summaries may be better in Random effect model approach compared to original treatments. This study has given a clear approach to Bayesian analysis which could be carried out on a fixed dataset relatively simple using MCMC to simulate posterior distributions. This study provides a direction to understand the recommendations given by the stock broker.

1. INTRODUCTION

Share market is the collective aggregation of buyers and sellers of stocks. It is a platform where people buy/sell their shares on the companies that are listed under the particular nation's share market. Shares are the individual units of ownership of the company. Generally when a company decides to list itself in the Share market, it releases some percentage of shares with the intention of bringing public investment into the company. Individuals who are interested investing in stock market in India,

has to approach stock exchanges. The most two prominent stock exchanges in India are - Bombay Stock Exchange (BSE) and National Stock Exchange (NSE).

For an individual it is not possible to buy the stocks direct from the exchange, so there is a need of an intermediate to continue with the process, such intermediate are called as stock brokers. A stock broker plays a major role in purchasing and reselling of stocks in the stock market, on behalf of its clients charging some fees or commission. A stock broker cannot start

their business without filing for appropriate registrations and obtaining certain memberships. They should also be able to provide a wide range of security information to clients for investment research and trade selections. According to Section 65(93) of the Finance Act of 1994, a stock broker is a person who has applied for or is registered as a stock broker in line with the Securities and Exchange Board of India Act, 1992's rules and regulations.

There are many stock brokerage firms in India through which a trade could be done in stocks. Based on the types of clients they cater to, Stock brokers are generally divided into two groups based on the sorts of clients they serve, Apart from stock trading, most brokers also provide services in the areas of bonds, mutual funds, futures, options, exchange traded funds, and commodities trading. Stock brokers can also provide investment advice for all of these items in addition to the services.

There were many studies related to this topic by various research scholars which includes behavioral pattern of the investors towards investment, capital market, secondary market, equity market, stock broking and stock brokers. A few literature works related to the study are, Srinivasan and Hanssens (2009) asserted that analysts' recommendations have significant impacts on firms sustainable performance because analysts' recommendations are more persuasive than other information channels for public investors due to their specialized reputation. Mohanraj and Kowsalya (2017) has done a study on the investor satisfaction towards service quality of stock brokers with reference to coimbatore District. Hou, Zhao, and Yang (2018) use all research reports produced on Shanghai Stock Exchange and Shenzhen Stock Exchange equities from 2008 to 2016 to build an information transmission network and study the link between analyst information connections and earnings predicting performance. Kalaiselvi and Sangeetha (2018) had studied on Ratio analysis of the selected stock broking companies.

Financial analysts may act as an information bridge linking brand equity and a firm's long-term success, according to Wang and Jiang (2019), who provide expert stock investment advice to public investors. Pan and Xu (2020) had examined whether analysts cash flow forecasts improve the profitability of their stock recommendations and whether the positive effect of cash flow forecasts on analysts stock recommendation performance varies with firms earnings quality. Sangeetha, C.(2018) has done the Analysis of the Financial Performance of a Stock broking Companies using multiple Regression. Su and Zhang (2021), a time-series bootstrapping simulation method to distinguish sell-side analysts skill from luck.

There are a limited research works that deal with the performance of stock brokers and their role on investment decisions. The goal is to gain a clear grasp of the factors stated in the data and to use a Bayesian statistical method to create a model based on the stock broker's recommendations. Association measures between the variables of interest could be achieved through odds ratio and finding the measure of heterogeneity is identified as the significant measure of this analysis. Generating the posterior distribution is obtained through MCMC which is basically using markov chains in which numerous complications can be solved using a common operating system. Andrieu et al., (2003) provides more details on the development and implementation of various MCMC techniques. STAN language (Gelman et al., (2017). Stan: a probabilistic programming language, has been used for Bayesian analysis in R. All the computations have been carried out in R which is freely available software package especially for statistics (Chambers, J. (2008)). A typical data set has been presented in section 2 with a detailed description of the variables; section 3 lists the methods and models applied in the paper; data treatment and interpretation has been presented in section 4; section 5 provides the discussions and conclusions derived from the analysis.

2. DATASET AND ITS DESCRIPTION

The data set which is considered in this study is collected from a stock brokerage firm which has been granted exclusively for academic purpose. They are involved in providing trading calls based on watchful research in technical and demonstrated model. The CUE app developed by them is a finest Equity market analytics and research platform which uses NSE and BSE data. With statistics and visualisation warnings, CUE is helpful. It is a piece of the mission of educating individual investors about the opportunities to build wealth in the equity market. This particular data is a typical rectangular data set which contains 6111 observations and 23 variables from the time period (January 2020 - December 2020). Whenever there exists a situation to

explore a data, interest lies in finding the variables involved in the process and the associations between the variables from which the uncertainty could be understood. On the other side interest lies in predicting the future events associated with the study. Such exercise involves a great confluence of background information or knowledge about the process and a science to learn from the data with an aid of a proper tool. It is important to prepare a meta data to understand the nature of the variables which is shown in Table 1.

The above table details the variable name, description, nature of variables and the number of levels each variable has within it. Out of 23 variables, 13 are categorical in nature. The first variable is the name of the stock which is

Table 1: Meta Data Explaining the Stock Data Considered in This Study

S. No	Variable Name	Description	Nature	Details
1	Stocks	Stock Name	Character	4 levels
2	Product_Type	Segment Codes	Factor	12 levels
3	Status_symbol	Call Sequence	Factor	Negative / Positive
4	Stk_OPN_Date	Date of a stock open	Character	Negative / Positive
5	Stk_CLS_Date	Date of a stock closed	Character	2 levels
6	Call_Amount	Outcome value - P/L in INR	Numeric	Positive
7	Net_Return	Outcome value - P/L after deduction in INR	Numeric	2 levels
8	Call_Status	Buy / Sell status when the call is initiated	Factor	7 levels
9	Entry_Rate	Rate when the call is initiated	Numeric	4 levels
10	Call_Result	Outcome status - P or L	Factor	1 to 12
11	Entry_Rate_C	Classification of Entry_Rate	Factor	4 levels
12	Segment	Name of the segment	Factor	5 levels
13	Month_Num	Number 1 - 12 corresponding to a month	Numeric	2 levels
14	Year	Year of the transaction	Character	2 levels
15	Quar	Quarter (Jan - Dec) of the transaction	Factor	2 levels
16	Outcome	Classification of Outcome	Factor	2 levels
17	Alerts	Whether the call has Alert	Factor	
18	Direct_Tgt	Whether the call yields Tgt2 directly	Factor	
19	Tgt1	Whether the call has Tgt1	Factor	
20	Net_Result	Outcome status - P or L	Factor	
21	St_date	Date of a stock open (Start)	Date	
22	Ed_date	Date of a stock closed (End)	Date	
23	Day_Diff	No of days a stock kept open	Numeric	

listed in NSE and BSE India. The product type refers the segment codes which represents four types of segments (1- cash, 2 – futures, 3 – options, 4 – index). The status symbol represents the call sequence which has 12 levels, which is then followed by the date when a stock is opened and then the date when a stock is closed. Call amount is the outcome value which could be profit or loss in nature. Net result is the outcome value which is calculated after deductions which also could be profit or loss.

Call status is either buy or sell when a call is initiated. Entry rate is fixed when the call is initiated which is metric in nature and is converted in to categorical variable using factor levels which is represented in new column Entry_Rate_C. Call result is the outcome value which could be profit or loss calculated before deductions and Net result is the outcome value which is calculated after deductions. Month number is specified in numbers ranging from (1 to 12) representing 12 months. The year of the transaction is mentioned as 2020 (January to December). Quarter represents four quarters in a year which ranges from Q1 to Q4.

Outcome is a variable with five levels representing, tgt: Target 1 rate, system would indicate; safe traders may decide to exit the stock. TrSL: Trailing Stop Loss rate indicating inverted U-turn in a bullish market. If after reaching T0 or T1, if the market tends to reverse then system will generate this message; safe traders may decide to exit the stock and system would close the call. SL: Stop loss rate; a limit for the bearish market. Exit_Profit & Exit_Loss: Indicates whether the call result is a profit or loss. Alert represents whether the particular call has an alert or not which is indicated as (0 or 1). Direct target indicates that whether the call has yielded target2 directly which has two factors (0 or 1) followed by target 1 whether the call had target1. Finally comes the start and the end date of a call and the number of days which the stock was kept open. The next chapter takes us

in identifying the major association metric and the principle to be adopted to carry out the analysis.

3. METHODS AND MATERIALS

The measure of association between the variables of interest could be estimated through odds ratio. It has become one of the increasingly important measure which helps in comparing two proportions. It is a simple analysis measure in 2 x 2 contingency tables. Agresti, A (2013) could be referred for more usefulness of odds ratio and its applications so as to acknowledge it as a preferred summary measure in random effect model in comparison of binomial proportions. The odds ratio, which is often used in medical research, can be a useful tool for determining if categorical projections and observations are related. Furthermore, significance tests on the logarithm of the odds ratio may be used to determine if the skill is entirely due to random sampling, as discussed in Stephenson, D. B. (2000). This research shows the concepts using Finley's classic set of tornado forecasts.

For a probability ϕ of a success, the odds are defined to be, $\rho = \frac{\phi}{(1-\phi)}$.

The ratio of the odds ρ_1 and ρ_2 in the two rows,

$$\Omega = \frac{\rho_1}{\rho_2} = \frac{\phi_1 / (1-\phi_1)}{\phi_2 / (1-\phi_2)}$$

is called the odds ratio.

For joint distributions with cell probabilities (ϕ_{ij}) the equivalent definition for the odds in row is

$$\rho_i = \phi_{i1} / \phi_{i2}, i = 1, 2$$

Now the odds ratio is,

$$\Omega = \frac{\phi_{11} / \phi_{12}}{\phi_{21} / \phi_{22}} = \frac{\phi_{11} / \phi_{22}}{\phi_{12} / \phi_{21}}$$

It is otherwise known as cross-product ratio, because it equals the ratio of the products $\phi_{11}\phi_{22}$ and $\phi_{12}\phi_{21}$ of probabilities from diagonally opposite cells. When the odds ratio is greater than 1, subject in the first row is more likely to have the first response than the subject in the second row. If the odds ratio lies between zero and one then the subject in the first row is less likely to have the first response than the subject

in the second row. The logarithm of the odds ratio (Log Odds) is often used as an alternate representation of probabilities in fields such as economics, biology, and artificial neural networks, and may be found in Pohl, K. M. et al., (2007).

Bayesian methods have been applied to problems in various fields like archaeology which could be referred in Buck, C. E., et al., (1996). Bayesian approach to econometrics could be referred in Dorfman, J. H. (1997). Bayesian statistics for evaluation research could be seen in Pollard, W. E. (1986). The interest lies in revealing the fundamental principle of the bayesian approach, where in the first step lies in assigning priors to all unknown parameters $p(\theta)$, then follows defining the likelihood of the data given the parameters $f(x/\theta)$ and finally determining the posterior distribution $\pi(\theta/x)$ of the parameters given the data. The practical approach to analysing data and solving research problems on Bayesian methods are clearly explained in Gelman, A., et al., (2013). Gelman, A., & Shalizi, C. R. (2013) explaining the philosophy and the practice of Bayesian statistics are essential but not complete list of sources. The other aspect of Bayesian procedure is the requirement for computations where in a remarkable growth could be seen in algorithms such as Markov Chain Monte Carlo (MCMC) Smith, B. J. (2007), Denwood, M. J. (2016), could be referred for implementation in R. Stan uses Markov chain Monte Carlo techniques to give complete Bayesian inference for continuous-variable models, as described in Carpenter, B., et al., (2017) and Gelman, A., et al., (2015).

Merging the information from multiple studies is one of the notable statistical application in psychology, academics, medical, human resource, ecology etc., De Maat, Saskia, et al. (2013). Shachar, M., & Neumann, Y. (2003). Richey, F., et al. (2004), Cawley, B. D., Keeping, L. M., & Levy, P. E. (1998), Nakagawa, S., & Poulin, R. (2012) are few to mention here. Though the objective and purpose may be different

among these methods, the underlying statistical model is random effects model. Random effect model is basically a hierarchical structure, mostly based on a normal distribution in two stages. Majority of the studies and even the most recent ones (Langan et al., 2017) follow normally distributed models using appropriate transformation of underlying parameters.

In Random Effect Model, if θ_i is an effect size estimate of a corresponding true effect size with the within-study variance σ_i^2 , then we could estimate θ_i , $i=1,2,\dots,k$ from the sample data; let us denote these estimated values as $\hat{\theta}_i$, $i=1,2,\dots,k$; that is $\hat{\theta}_i$, $i=1,2,\dots,k$. We calculate the Odds Ratio for the two rows using an independent binomial distribution. Precisely for the two rows in each table,

$$X_1 \sim Bin(n_1, \theta_1)$$

$$X_2 \sim Bin(n_2, \theta_2)$$

Then we define $\Omega = \log it(\theta_1) - \log it(\theta_2)$

$= \log \left(\frac{\theta_1 / (1 - \theta_1)}{\theta_2 / (1 - \theta_2)} \right)$ which is log odds ratio, the quantity of interest in log scale for each of the k tables.

Also, for k tables we define,

$$\mu = \frac{\log it(\theta_1) + \log it(\theta_2)}{2}$$

Hence the second stage of the model is specifying priors for μ and Ω

$$\mu \sim N(\mu_0, \sigma_0^2) \quad (1)$$

$$\Omega \sim N(d, \tau^2) \quad (2)$$

σ_0^2 amounts sampling variability in the effect size θ_1 estimate. τ^2 amounts variability between the effect size (among the grouping or stratifying variables). The final summary would help to clarify the relationship between the variables on an individual and general level, as well as the degree of heterogeneity.

4. DATA TREATMENT AND INTERPRETATION

The principle objective lies in proposing a model or a quality indicator for stock broker

based on their call recommendations. Dataset for one complete year (2020) has been received from the stock brokerage firm. The various levels involved in data treatment are given as follows:

The format for this analysis is a two-fold contingency table that classifies two dichotomous variables. In this case, two levels of X_1 are Buy, sell with regard to call status and that of X_2 are Profit, Loss with regard to call result. Each cell count is the number that accounts for the respective combination of X_1 and X_2 . The stratifying variables are month-wise and Entry rate.

Transforming the rectangular dataset which is received from the brokerage in to a 2 x 2 data set.

Table 2: Data format for Month-wise and Entry Rate wise in the year 2020

X_1/X_2	Profit	Loss
Buy	n_{11}	n_{12}
Sell	n_{21}	n_{22}

Random effect model is considered as the underlying model in order to understand the association between the variables of interest and the variability quantification.

Two typical models have been identified for this study of which the first model is the month-wise categorization (12 levels) and then followed by Entry rate (price) categorization (7 levels). Then Entry rate is a metric variable and it is treated as a categorical variable by assigning levels to it. Then the next step lies in identifying the suitable associated variables to get two more dimensions. The details of the variables identified in this present study are clearly explained below.

After the data is treated and identified the right suitable associated variables the final output will be in the form of a k x 2 x 2 data set and it is extracted for both the models. This study has focussed to make use of the benefits of Bayesian modelling in handling hierarchical

Table 3: Details of variables considered in this study.

Model	Grouping variable	Predictor variable	Response variable
M1	Month-wise	Call Status	Call Result
M2	Entry Rate	Call Status	Call Result

models. The whole interest lies in drawing the posterior distributions through appropriate algorithm. Bayesian analysis for intricate models can be made quite simple using MCMC to simulate posterior distributions. Basically, a Markov chain works on a simple principle of future depends only on the present, not on the past. So essential needs for MCMC are where to start, when to stop, how to allow them to mix and most important feature lies in picking independent samples. In this study, the initial step is to determine the,

Number of chains: C=4

Number of simulations: N=20000

Number of burn-ins: B=3000

Thin, period for saving samples: T=10.

Then finally, generated samples will have
 Number of samples = $C \times \frac{(N - B)}{T}$

This study includes a normal model for a log transformed Binomial parameter. The models are presented in a schematic way as given below $X_1 \sim Bin(n_1, \theta_1)$

Scheme I: $X_2 \sim Bin(n_2, \theta_2)$

Then defining, $\Omega = \log it(\theta_1) - \log it(\theta_2)$ which is log-odds ratio. For k tables, we define

$$\mu = \frac{\log it(\theta_1) + \log it(\theta_2)}{2}$$

$$\theta_1 = \log it^{-1}(\mu + \frac{\Omega}{2})$$

$$\theta_2 = \log it^{-1}(\mu - \frac{\Omega}{2})$$

Scheme II: $\mu \sim N(\mu_0, \sigma_0^2)$ (1)

$$\Omega \sim N(d, \tau^2)$$
 (2)

Scheme II: $d \sim N(m_d, S_d)$

$$\tau^2 \sim \text{inv gamma}(\tau_1, \tau_2)$$

All hyper parameters $\mu_0, \sigma_0^2, \tau_1, \tau_2, m_d, s_d$ are appropriately chosen and the choices are presented as for Scheme II, $N(0, 10^4)$ are for mean parameter and in Scheme III, the hyper priors in inverse gamma distribution is retained with (3,1).

The entire exercise has been carried out in R and R-Stan has been used to do Bayesian analysis. The final summary measures are presented in the Table. Point and confidence interval estimates for individual and overall Odds Ratio together with the measure of heterogeneity (τ^2) for Profit or loss in month – wise and Entry Rate analysis.

Table 4: Combined Odds Ratio for month wise analysis; LL and UL are lower and upper limits of 97.5% confidence interval.

Month	Estimate	LL	UL
Jan	0.703	0.545	0.910
Feb	0.859	0.649	1.140
Mar	0.964	0.638	1.470
Apr	1.422	0.975	2.066
May	0.892	0.636	1.237
Jun	0.669	0.503	0.895
Jul	0.750	0.506	1.100
Aug	0.647	0.434	0.972
Sep	0.719	0.475	1.081
Oct	0.979	0.646	1.467
Nov	1.215	0.790	1.877
Dec	0.874	0.622	1.237
Overall	0.870	0.659	1.142
Heterogeneity	0.200	0.094	0.400

From Table 4, it is observed that individual and overall OR is less than 1 except for the month April and November which indicates that the odds, when a stock is sold and resulting in profit is always higher than when a stock is bought and yield a profit. In the month of April and November the odds are higher for the case, stock is bought and the resultant is profit. Also, the estimates are statistically significant only in the month of January, June and August at $\alpha = 2.5\%$ with the length of confidence interval is maximum in the month of April. The overall OR estimate is less than 1 indicating that in the year

Table 5: Combined Odds Ratio for month wise analysis; LL and UL are lower and upper limits of 97.5% confidence interval.

Entry Rate	Estimate	LL	UL
<=500	1.200	0.654	2.208
501-1500	0.678	0.556	0.826
1501-2500	0.888	0.737	1.072
2501-3500	0.879	0.662	1.168
3501-4500	0.733	0.540	0.997
4501-5500	1.199	0.726	1.996
>5500	0.835	0.597	1.162
Overall	0.900	0.596	1.361
Heterogeneity	0.250	0.104	0.575

2020 the odds are in favour of resulting in a profit only when a stock is sold than yielding a profit when it is bought. Further a positive measure of heterogeneity which is 20% with notable wider 97.5% confidence intervals indicate a heterogeneous effect size across twelve months.

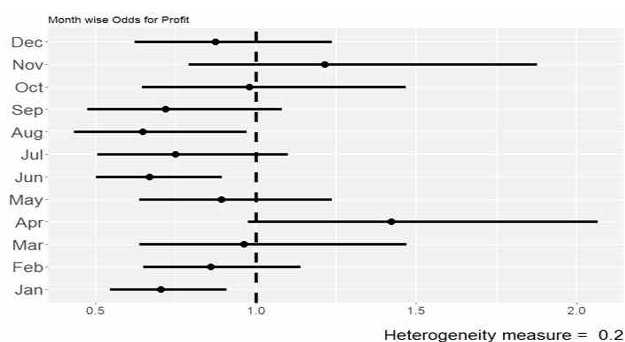
From Table 5, it is observed that the odds ratio is less than 1 except for the entry rate <=500 and 4501-5500 which indicates that the odds are in favour when selling a stock which gives a profit rather than when a stock is bought and it results in profit. The overall OR measure is less than 1 indicating that odds are higher for the stock when sold and yields a profit rather than buying a stock which result in profit. The estimates are statistically significant only in the entry rates 501-1500 and 3501-4500. Further a positive measure of heterogeneity which is 25% indicates a heterogeneous effect size across different entry rates.

Individual measure such as month wise and entry rate wise specific odds ratio (θ_i) together with 97.5% credible intervals is presented in forest plot (Shim, S. R., & Kim, S. J. (2019)) for easier visual representation and better understanding of variability. Figures 1 and 2 are the forest plot for the month-wise and entry rate wise analysis respectively.

Figure 1, shows the call result ending in profit when a stock is sold rather than when it is bought and the estimates are statistically

significant only in the month of January, June and August. The intervals are much wider in the month of April and November. In Figure 2, it is clear from the forest plot that the estimate is statistically significant in the levels 501-1500 and 3501-4500. A much wider notable credible intervals could be seen in the category ≤ 500 and 4501-5500, the odds are higher for the resultant to be profit in the case when a stock is sold

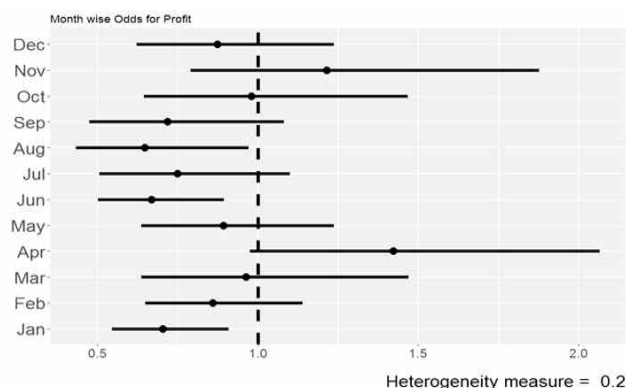
Figure 1 : Forest plot of the point and interval estimates of individual odds ratio for the result to be a profit corresponding to twelve months in the year 2020.



5. CONCLUSION

This study has made an attempt in building a model based on the recommendations of a stock broker. Based on the data received from the stock broker, the

Figure 2 : Forest plot of the point and interval estimates of individual odds ratio for the result to be a profit corresponding to various entry rate category wise.



response metric variable is treated as a categorical variable using appropriate rules. Identifying suitable associated variables to understand the variability quantification in a more better way and the summaries may be better in Random effect model approach compared to original treatments. This study has given a clear approach to Bayesian analysis which could be carried out on a fixed dataset relatively simple using MCMC to simulate posterior distributions. This study provides a direction to understand the recommendations given by the stock broker.

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